To whom it may concern

By Frank R. Greening
Physical Chemistry
Hamilton, Ontario, Canada
greening@sympatico.ca

The Journal of 911 Studies has recently published a paper entitled *Momentum Transfer Analysis of the Collapse of the Upper Storeys of WTC 1* by Gordon Ross. The *Journal* paper presents an alternative approach to an earlier treatment of this topic by F. R. Greening: *Energy Transfer in the WTC Collapse*, (available in pdf format at 911myths.com). The paper by Ross claims to show that a simple gravitationally driven collapse of the upper section of WTC 1 would have been arrested after the first impact. By contrast, Greening's previously mentioned paper concludes that a self-sustaining progressive collapse was physically possible for WTC 1 & 2.

I intend to show that *Mr. Ross' article is incorrect in at least four important ways*:

(i) An error in the calculation of the kinetic energy of the falling section after the first impact.

(ii) An over-estimation of the energy required to pulverize the WTC 1 concrete.

(iii) The argument that the initial elastic deflection would propagate 24 storeys below the impact floor.

(iv) The use of a safety factor of 4 in the calculation of the elastic strain energy.

We now consider these items in detail:

1. The Kinetic Energy

On page 4 of the article by Ross we read:

"To estimate and illustrate the … momentum changes we can assume that the storey which is 25 storeys from the impact remains static and the velocity of the 24 affected storeys will vary linearly from the velocity of the falling section to zero.

Momentum before impact = 16 storeys moving at 8.5 m/sec

*Momentum after impact = 17storeys moving at V2 m/sec + 1 storey moving at 23/24*V2 m/sec + 1 storey moving at 22/24*V2m/sec ……+ 1 storey moving at 2/24*V2 m/sec + 1 storey moving at 1/24*V2m/sec*

*16*8.5 = V2 (17 + 11.5)*

*V2 = 16 * 8.5 / 28.5 = 4.8 metres per second.*

The speed of the upper section would be reduced by the collision from 8.5 m/sec to a speed of less than 4.8 m/sec rather than the 8 m/sec derived from a momentum calculation which does not include this factor.

The kinetic energy of the falling section would be similarly affected, but because of the velocity squared relationship, the reduction in kinetic energy would be more pronounced.

K. E. of falling section before impact = 16 floors moving at (8.5 m/sec)^2

K. E. of falling section after impact = 17 floors moving at (4.8 m/sec)^2

*Percentage loss of K.E. = 1- (17 * 4.8^2) / (16 *8.5^2) * 100% = 66%*"

It is in these last lines that we discover a serious error in Mr. Ross' calculation. In particular, the kinetic energy of the falling upper section *after impact* is NOT "*17 floors moving at (4.8 m/sec)^2*". Why? Because Mr. Ross has just told us that the motion after impact involves:

"*17storeys moving at V2 m/sec + 1 storey moving at 23/24*V2 m/sec + 1 storey moving at 22/24*V2m/sec ......+ 1 storey moving at 2/24*V2 m/sec + 1 storey moving at 1/24*V2m/sec*"

THIS IS AN EFFECTIVE MASS THAT IS MUCH LARGER THAN 17 STOREYS

It is a simple matter to calculate what the post-collision effective mass should be using the well-known momentum conservation relationship:

$$M_1 V_1 = M_2 V_2$$

Using Ross' figures we have:

$$V_1 = 8.5 \text{ m/s}^2 \text{ and } V_2 = 4.8 \text{ m/s}^2$$

Hence,

$$M_2 = M_1 \times [8.5 / 4.8] = 1.77 M_1$$

With this mass we may now re-calculate the KE loss to be 1182.5 MJ rather than Ross' incorrect figure of 1389 MJ, a difference of about 206 MJ in favor of sustained collapse.

2. The Energy to Pulverize Concrete

We now consider the energy required to crush or pulverize the 4-inch (10 cm) layer of concrete on each WTC Tower floor. Consider the example of the WTC 1 collapse involving the descent of 16 blocks of floors a distance of 3.7 meters onto the floor below. After the initial impact, the loading of the lower floor concrete increased for a finite time interval we shall call Δt, at which point the yield strength of the concrete was reached and the concrete failed by brittle fracture. For simplicity we shall assume that the compressive force acting on the concrete increases linearly up to a value F_y given by,

$$F_y = \sigma_y A$$

where σ_y is the *effective* yield strength of WTC concrete in N/m^2, and A is its surface area in m^2. Thus the compressive force acting on the concrete has an *average* intensity of ½ F_y Newtons for a time Δt, and imparts a change of momentum to the falling mass of 16 floors given by:

$$M_{16} \Delta v = ½ F_y \Delta t$$

In which case:

$$\Delta v = (\sigma_y A \Delta t) / 2 M_{16}$$

The quantity Δv is a measure of the loss in kinetic energy of the falling block of floors, which in turn is equivalent to the energy, E_c, expended in fracturing and crushing the concrete on one floor. This may be expressed mathematically as:

$$E_c = ½ M_{16} v^2 - ½ M_{16} (v - \Delta v)^2$$

where v is the impact velocity.

If Δv is small compared to v it follows that to a good approximation:

$$E_c = M_{16} v \Delta v$$

hence, substituting for Δv, and solving the equation we have

$$E_c = ½ v (\sigma_y A \Delta t)$$

In order to proceed further with this analysis we note that experimental values of Δt for the fracture of concrete are available in the published literature, /1, 2, 3/. The published values fall in the range 0.5 – 5 milliseconds, hence we will take 3 ms as a reasonable value for Δt. Also, for the already damaged concrete on the critical WTC floors we will take σ_y to be 5 MN/m^2 and A to be 2000 m^2. Hence,

$$E_c = ½ v \times 5 \times 10^6 \times 2000 \times 3 \times 10^{-3} \text{ Joules}$$

or,

$$E_c = v (1.5 \times 10^7) \text{ Joules}$$

As an example of the use of this equation we note that the velocity of the first impact in WTC 1 is 8.5 m/s, in which case E_c = 128 MJ a value that is 176 MJ <u>smaller</u> than the estimate of 304 MJ made by Mr. Ross, which in turn is based on Greening's <u>estimate</u> in his *Energy Transfer in the WTC Collapse* report.

How do we account for this difference? Ironically Ross does not offer his own method of calculating the energy to crush the WTC concrete but relies on a simplified approach first proposed by Greening in his *Energy Transfer in the WTC Collapse* report. We note that Greening's 2005 report assumed *as a first approximation* that the concrete was pulverized to a constant particle size for each and every floor. Our new and improved formalism shows that the particle diameter is in fact proportional to 1/v, or equivalently, the specific surface *area* of the particles is proportional to v^2. Now, since the impact energy is ½ $M_{16} v^2$, we have the physically appealing result that the specific surface area of the particles increases as the impact energy increases, a behavior reported for rock fragmentation under high speed impact loading /4, 5/.

3. The argument that the initial elastic deflection would propagate 24 storeys below the impact floor.

An examination of the many available videos of the collapse of WTC 1 & 2 shows that the blocks of floors above the aircraft impact zones did not simply descend vertically onto the floors below but tipped over as they fell. In Chapter 2 of the FEMA Report on the World Trade Center Disaster we read:

"A review of aerial photography of the site, following the collapse, as well as identification of pieces of structural steel from WTC 2, strongly suggests that the top portion of the tower fell to the south and east, striking Liberty Street and the Bankers Trust building."

A discussion of the initiation of the asymmetric collapse of the upper section of WTC 2 is given in the NIST Final Report on page 308 of Chapter 9 of NISTSTAR 1-6 where we read in reference to the condition of WTC 2 just moments before it began to collapse:

"The entire section of the building above the impact zone began tilting as a rigid block (all four faces; not only the bowed and buckled east face) to the east (about 7° to 8°) and south (about 3° to 4°) as column instability progressed rapidly from the east wall along the adjacent north and south walls."

WTC 1's collapse also involved a tilting of the upper section of the Tower and was therefore asymmetric. Thus the downward collapsing force had a significant *angular* component. Why is this important? Because the longitudinal compression wave induced by the initial rotational (tilting) action and free fall collapse of the upper sections of WTC 1 & 2 was <u>*not*</u> propagated down the central vertical axis of the columns. Lateral and even torsional compression waves were created. This means that most of the initial impact kinetic energy was expended in destroying the first impacted floor as proposed in Greening's *Energy Transfer in the WTC Collapse* report. In addition it is well known that an elastic compression wave in a spliced column system such as the WTC will <u>*not*</u> propagate efficiently, but dissipate, at each splice. Thus there is no justification for the assumption that the initial elastic deflection would propagate 24 storeys below the impact floor. This is an idealized concept *that was not satisfied in the collapse of the Twin Towers*.

But were there any *visual indications* of how the impact of the upper block of WTC 1 affected the lower portion of the building? Certainly, if Ross' suggestion that 24 floors below about the 95th floor moved downward after the impact of the upper section, the videos and photographs of the collapse of WTC should show a noticeable downward displacement of floors between the 70th and 94th levels immediately after the impact. In fact, no such movement was observed. It is significant, however, that ejections of dust and debris *were observed* at a few locations several floors below the impacted floors. This observation suggests that the fast moving compression wave did inflict some damage to floors a few storeys below the directly impacted floors. However, *and this is an important point*, such "pre-damage" to lower floors should not be considered as "lost" kinetic energy but rather as energy that facilitated the later total collapse of the affected floors.

Thus we suggest that, instead of 24 floors, *a maximum* of four floors would have shown any significant downward movement after impact of the upper block of floors. We therefore re-calculate Ross' momentum loss term (using his formalism) as follows:

Momentum after impact = 17 storeys moving at V_2 m/s + 1 storey moving at 4/5 V_2 m/s + 1 storey moving at 3/5 V_2 m/s + 1 storey moving at 2/5 V_2 m/s + 1 storey moving at 1/5 V_2 m/s

From which we find that:

$$16 \times 8.5 = V_2 [17 + 2]$$

or,

$$V_2 = (16 \times 8.5) / 19 = 7.16 \text{ m/s}$$

This implies that Ross' post impact kinetic energy is 1580 MJ so that the KE "loss" is only 525 MJ, or 864 MJ less than Ross' figure.

4. The use of a safety factor of 4 in the calculation of the elastic strain energy

On pages 6 and 7 of Ross' article we see an undefined "safety factor", arbitrarily set at 4, used to calculate the elastic strain energy of the lower and upper storeys. In looking for any justification for the use of a safety factor of 4 for the WTC we read in Reference /6/:
" *The factor of safety is typically not greater than 2 in building structural designs*."
(Note added July 19th, 2006: S. Sunder at a NIST Progress Report on the *WTC Building Performance,* presented Oct 19th, 2004, stated that the safety factor for the yielding and buckling of core columns is 1.67.)

If Ross' calculation of the elastic strain energy is re-done with a safety factor of 2 instead of 4, the claimed *energy deficit* must be decreased by 95 MJ.

Conclusions

We have identified four problems with the momentum transfer calculations of Ross: First we have an error in the calculation of the kinetic energy of the falling section after the first impact. Second, we have an over-estimation of the energy required to pulverize the WTC 1 concrete. Third, we show that the assumption that the initial elastic deflection would propagate 24 storeys below the impact floor is physically unrealistic and should be revised to a maximum of 4 storeys. And finally we argue why a safety factor of 2, rather than 4, is appropriate for the calculation of the elastic strain energy.

The error in the determination of the kinetic energy removes 206 MJ from Ross' calculation of the energy losses during the first impact of the upper section of WTC 1. The correction of the overestimation of the energy needed to pulverize the WTC 1 concrete, which Ross includes for the impacted and impacting floors, removes 2 times 176 MJ or an additional 352 MJ from Ross' energy deficit. Our revision to the elastic deflection term and the elastic strain energy remove 864 MJ and 95 MJ, respectively, from Ross' energy deficit. Thus we need to correct Ross' claimed energy deficit of –390 MJ by + 1517 MJ giving an energy excess of +1127 MJ, confirming that a gravity driven collapse of WTC 1 was in fact sustainable.

REFERENCES

1. S. Mindess et al. "*A Preliminary Study of the Fracture of Concrete Beams Under Impact Loading Using High Speed Photography*." Cement and Concrete Research 15, 474 (1985)
2. V. S. Gopalaratnam et al. "*A Modified Instrumented Charpy Test for Cement-Based Composites*.' Experimental Mechanics 24, 102 (1984)
3. C. T. Yu et al. "*Energy Dissipation Mechanisms Associated with Rapid Fracture of Concrete*." Experimental Mechanics 33, 205 (1993)
4. Y. Yalun. "*Characteristics of Rock Under High-Speed Impact Loading*." Proceedings of the International Symposium on Intense Dynamic Loading and its Effects. Pages 820 – 825, Pergamon Press (1988)
5. F. Ouchterlony. "*The Swebrec Function: Linking Fragmentation by Blasting and Crushing*." Mining Technology 114 A29 (2005)
6. Q. Zhou et al "*Use of High-Efficiency Energy Absorbing Device to Arrest Progressive Collapse of Tall Buildings*" Journal of Engineering Mechanics, 130(10), pp 1177 – 1187 , (2004)